# EFFECT OF VARIABILITY OF PHYSICAL PROPERTIES OF A GAS ON TURBULENT FLOW AND HEAT TRANSFER IN A PIPE WITH PERMEABLE WALLS

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We have calculated turbulent quasideveloped flow and heat transfer in a circular pipe, taking account of the temperature dependence of the physical properties of the gas when there is injection through porous walls.

The combined effect of injection and nonisothermicity on friction and heat transfer in the entrance region of a pipe was treated in [1, 2]; it was found that the laws are the same as in a turbulent boundary layer on a plate [3]. There has been practically no systematic research on the effect of the variability of the physical properties in pipes with permeable walls for quasideveloped flow far from the entrance section. Only in [4] were results presented of a calculation of the Nusselt number for the flow of helium in the near-critical region of the state parameters for an "improved" heat-transfer regime, from which it was concluded that the effect of the variability of the properties on heat transfer decreases with increasing rate of suction or injection.

In resolving the velocity and other variables into average and pulsating components in the analysis of turbulent flow with variable properties, it is expedient to use the density as the weighting function in performing the averaging [5]. In this case $\langle \rho' u_i' \rangle = \langle \rho' h' \rangle = 0$ and the average equations of motion and heat transfer are written in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_k)}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial}{\partial x_k} (\rho u_i u_k + p \delta_{ik} + \langle \rho u_i' u_k' \rangle - \sigma_{ik}) = 0, \tag{2}$$

$$\frac{\partial (\rho h)}{\partial t} + \frac{\partial}{\partial x_k} \left( \rho h u_k + \langle \rho h' u_k' \rangle - \lambda \frac{\partial T}{\partial x_k} \right) = 0, \tag{3}$$

where

$$\sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right).$$

The equation for the Reynolds stress $\langle \rho u_i' u_j' \rangle$ has the form

$$\frac{\partial \langle \rho u_i' u_j' \rangle}{\partial t} + \frac{\partial}{\partial x_k} [ \langle \rho u_i' u_j' \rangle u_k + \langle \rho u_i' u_j' u_k' \rangle + ( \langle p' u_i' \rangle \delta_{jk} + \langle p' u_j' \rangle \delta_{ik}) - ( \langle u_i' \sigma_{jk}' \rangle + \langle u_j' \sigma_{ik}' \rangle )] = \left\langle p' \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right) \right\rangle - \left( \left\langle \sigma_{ik}' \frac{\partial u_j'}{\partial x_k} \right\rangle + \left\langle \sigma_{jk} \frac{\partial u_i'}{\partial x_k} \right\rangle \right) - \left( \langle \rho u_i' u_k' \rangle \frac{\partial u_j}{\partial x_k} + \langle \rho u_j' u_k' \rangle \frac{\partial u_i}{\partial x_k} \right), \tag{4}$$

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i.e., it is essentially the same as the equation for constant physical properties [6].

In Eq. (4) we set $\langle \rho u_i' u_j' \rangle \approx \rho \langle u_i' u_j' \rangle$, and assume

$$\frac{\partial}{\partial x_k}\left(\langle u_i' \sigma_{jk}' \rangle + \langle u_j' \sigma_{ik}' \rangle\right) - \left(\left\langle \sigma_{ik}' \frac{\partial u_j'}{\partial x_k}\right\rangle + \left\langle \sigma_{jk}' \frac{\partial u_i'}{\partial x_k}\right\rangle\right) \approx \frac{\partial}{\partial x_k}\left(\mu \frac{\partial \langle u_i' u_j' \rangle}{\partial x_k}\right) - 2\mu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}\right\rangle . \quad (5)$$

We determine the exchange and dissipative terms in Eq. (4) by using the approximate Rotta relations [7]

$$\left\langle p' \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)\right\rangle = -k \frac{\rho E^{1/2}}{L}\left(\langle u_i' u_j' \rangle - \frac{2}{3} E\delta_{ij}\right), \quad (6)$$

$$2\mu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}\right\rangle = \frac{2}{3} c \frac{\rho E^{3/2}}{L} + c_1 \frac{\mu \langle u_i' u_j' \rangle}{L^2}, \quad (7)$$

and represent terms describing turbulent diffusion in the simplest gradient form

$$\langle \rho u_i' u_j' u_k' \rangle + \langle p' u_i' \rangle \delta_{jk} + \langle p' u_j' \rangle \delta_{ik} = -\alpha_1 \rho E^{1/2} L \frac{\partial \langle u_i u_j \rangle}{\partial x_k}. \quad (8)$$

Using Eqs. (5)-(8), we obtain from (4) the equation for the intensity of velocity pulsations (turbulent energy) $E = \langle u_k' u_k' \rangle/2$:

$$\rho \frac{\partial E}{\partial t} + \rho u_k \frac{\partial E}{\partial x_k} = \frac{\partial}{\partial x_k}\left[(\mu + \alpha \rho E^{1/2} L) \frac{\partial E}{\partial x_k}\right] - c \frac{\rho E^{3/2}}{L} - c_1 \frac{\mu E}{L^2} - \rho \langle u_i' u_k' \rangle \frac{\partial u_i}{\partial x_k}. \quad (9)$$

We consider stationary quasideveloped flow of an incompressible gas in a circular pipe far from the entrance section. To construct the solution of Eqs. (1)-(3), (9) in the region of quasideveloped flow, we use the approximate method of separation of variables (method of local similarity) [8-10], in accord with which we assume

$$u_x = \bar{u}_x(\bar{r}) \rho \bar{U}/\rho_w, \; u_r = \bar{u}_r(\bar{r}) V_w, \; T = \bar{T}(\bar{r}) T_w, \; E = \bar{E}(\bar{r}) \rho \bar{U}^2/\rho_w^2,$$

$$\frac{\partial U_x}{\partial x} = \frac{\rho_m}{\rho_w} \frac{dU_m}{dx} \bar{u}_x, \quad \frac{\partial h}{\partial x} = \frac{2q_w}{\rho \bar{U} r_0}, \quad \frac{\partial E}{\partial x} = \frac{2U_m \rho_m^2}{\rho_w^2} \frac{dU_m}{dx} E. \quad (10)$$

The variation of the average velocity $U_m = \rho\bar{U}/\rho_m$ along the length of the pipe is determined from the relation

$$\frac{dU_m}{dx} = -\frac{2\rho_w V_w}{\rho_m r_0} + \frac{2q_w}{c_{p_m} T_m \rho_m r_0}, \quad (11)$$

where the first term describes the variation of the average velocity of flow as a result of injection, and the second as a result of the variation of the density during heating or cooling (thermal acceleration or retardation of the flow).

By using Eqs. (10) and (11), the system of partial differential equations (1)-(3), (9) is reduced to ordinary differential equations in dimensionless form:

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}}(\bar{r} \bar{\rho} \bar{u}_r) - 2\bar{\rho} \bar{u}_x = 0, \quad (12)$$

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}}\left[\bar{r}(\bar{\mu} + \bar{\mu}_t) \frac{d\bar{u}_x}{d\bar{r}}\right] - R\bar{\rho}\bar{u}_r \frac{d\bar{u}_x}{d\bar{r}} + (2R - K_c \Theta_w Q) \bar{\rho} \bar{u}_x^2 + K_p = 0, \quad (13)$$

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}}\left[\bar{r}(\bar{\lambda} + \bar{\lambda}_t) \frac{d\bar{T}}{d\bar{r}}\right] - R \,\mathrm{Pr}_w \bar{c}_p \bar{\rho} \bar{u}_r \frac{d\bar{T}}{d\bar{r}} - \mathrm{Pr}_w Q \bar{\rho} \bar{u}_x = 0, \quad (14)$$

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}}\left[\bar{r}\left(\bar{\mu} + \frac{\alpha_1 \mathrm{Re}_w}{2} \bar{\rho} \bar{E}^{1/2} \bar{L}\right) \frac{d\bar{E}}{d\bar{r}}\right] - R\bar{\rho}\bar{u}_r \frac{d\bar{E}}{d\bar{r}} +$$

$$+ 2(2R - K_c \Theta_w Q) \bar{\rho} \bar{u}_x \bar{E} - \frac{c \,\mathrm{Re}_w}{2} \frac{\bar{\rho} \bar{E}^{1/2}}{\bar{L}} - c_1 \frac{\bar{\mu} \bar{E}}{\bar{L}^2} + \bar{\mu}_t \left(\frac{d\bar{u}_x}{d\bar{r}}\right)^2 = 0. \quad (15)$$

Since the flows considered are close to self-similar, the mechanism of turbulent transport is described by a simple one-parameter model of turbulence [10] consisting of the equation of balance of pulsational energy (15), the Prandtl–Nikuradse formulas for the scale of turbulence (length of the displacement path)

$$\bar{L} = L/r_0 = 0{,}14 - 0{,}08\bar{r}^2 - 0{,}06\bar{r}^4$$

and the expression for the coefficient of the turbulent viscosity

$$\bar{\mu}_t = \mu_t/\mu_w = \alpha\,\mathrm{Re}_E/(1+\beta/\mathrm{Re}_E),\quad \mathrm{Re}_E = \rho E^{1/2}L/\mu_w. \tag{16}$$

The constants in (15) and (16) have the same values as for constant physical properties [10]; $c = 0.13$, $c_1 = 0.32$, $\alpha_1 = 0.2$, $\alpha = 0.51$, $\beta = 14$. The turbulent Prandtl number is assumed constant ($Pr_t = 0.9$), i.e, the turbulent thermal conductivity is determined by the formula $\bar{\lambda}_t = \lambda_t/\lambda_w = Pr_w c_p \bar{\mu}_t/Pr_t$.

The boundary conditions for Eqs. (12)-(15) have the form

$$\bar{r}=0\ \bar{u}_r = \frac{d\bar{u}_x}{d\bar{r}} = \frac{d\bar{T}}{d\bar{r}} = \frac{d\bar{E}}{d\bar{r}} = 0;\ \bar{r}=1\ \bar{u}_x = \bar{E} = 0,\ \ \bar{u}_r = \bar{T} = 1.$$

Calculations were performed for air ($Pr = 0.7$) whose physical properties vary with temperature according to the power laws

$$\bar{\rho} = \bar{T}^{-1},\ \bar{\mu} = \bar{T}^{0,7},\ \bar{\lambda} = \bar{T}^{0,8},\ \bar{c}_p = \bar{T}^{0,1}.$$

Figure 1 shows the effect of the variability of the properties and injection on the distribution of velocity, temperature, and turbulent energy over the cross section of the pipe. If there is no injection, the effect of the variability of the properties on the velocity and temperature profiles is not very large; while the population of the velocity profile is increased during heating and decreased during cooling, the population of the temperature profile, on the contrary, is increased during cooling and decreased during heating. These results agree with experimental data in [11]. The opposite character of the effect of the variability of the physical properties on the velocity and temperature profiles is preserved also during injection, but becomes appreciably more pronounced. A characteristic feature of the velocity distribution during injection, as for laminar flow [9], is the displacement of the maximum velocity from the pipe axis toward the wall with an increase in the heating rate.

If there is no injection, the rate of velocity pulsations is increased in the flow core and decreased near the wall during cooling, while during heating, on the other hand, the flow is turbulent near the wall and laminar in the flow core. A similar effect of cooling and heating on the distribution of turbulent energy over the cross section of a pipe was obtained in [12]. If there is injection, the effect of a change of properties on the intensity of velocity pulsations on the whole remains the same as for $V_w = 0$.

Figure 2 shows the calculated values of the coefficient of friction $\xi_m = 16\,\rho_m/\rho_w\,\mathrm{Re}_w\,(d\bar{u}_x/d\bar{y})_{\bar{y}=0}$ (solid curves) and the Nusselt number $\mathrm{Nu}_m = 2\,\Theta_w\lambda_w/\lambda_m(1-\Theta_w)\,(d\bar{T}/d\bar{y})_{y=0}$ (dashed curves) relative to their values without injection, for constant physical properties and the same Reynolds number $Re_m$. It is clear that in the absence and presence of injection the variability of the properties has a greater effect on heat transfer than on frictional drag, and in addition is substantially more pronounced during heating than during cooling. These conclusions are confirmed by experiment [13] for a pipe with impermeable walls. For comparison the figure shows the empirical dependences of the relative coefficients of friction and heat transfer on the temperature factor for $V_w=0$: $\xi_m/\xi_{0m}=\Theta_w^{-0,16}$ [14] (curve 5) and $\mathrm{Nu}_m/\mathrm{Nu}_{0m}=\Theta_w^{-0,5}$ (curve 6). It can also be seen from Fig. 2 that while the effect of the variability of properties on heat transfer is unique for all injections in the case of heating, and lowers the Nusselt number, the coefficient of friction for strong injection may increase with an increase of the temperature factor. This kind of change of the coefficient of friction, as well as the effect of the displacement of the maximum velocity from the axis toward the wall, is accounted for by the acceleration of the heat light gas in the region near the wall under the action of a negative pressure gradient induced by injection.

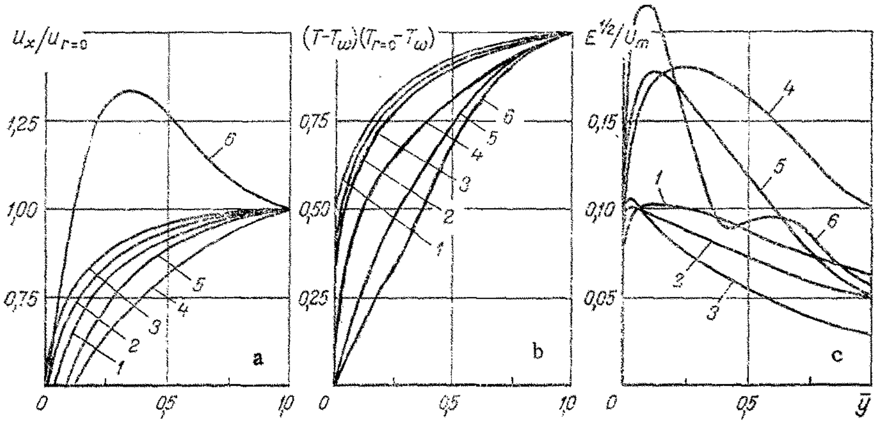


Fig. 1. Profiles of a) velocity; b) temperature; c) turbulent energy for $Re_w = 5 \times 10^4$; 1) M = 0 and $\theta_w = 0.5$; 2) 0 and 1; 3) 0 and 2; 4) −0.02 and 0.5; 5) −0.02 and 1; 6) −0.02 and 2.

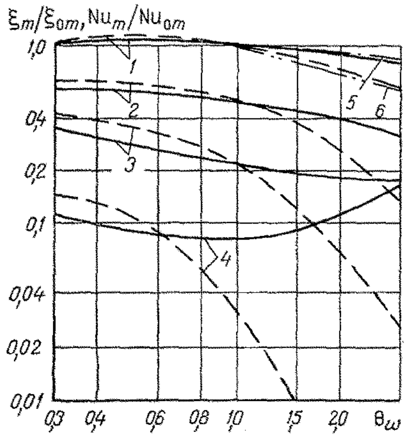


Fig. 2

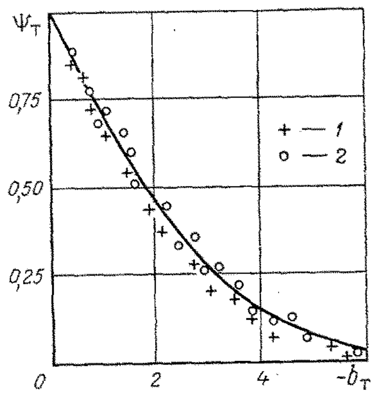


Fig. 3

Fig. 2. Coefficients of friction and heat transfer ($Re_w = 5 \times 10^4$): 1) M = 0; 2) −0.005; 3) −0.01; 4) −0.02.

Fig. 3. $\Psi_T(b_T)$ for 1) heating and 2) cooling.

The values calculated for the heat-transfer coefficient are satisfactorily summarized in the form of a relative heat-transfer law $\Psi_T = f(b_T)$. Here

$$\Psi_T = Nu_m(Re_m,\ Pr,\ \Theta_w,\ Pe_v)/Nu_{0m}(Re_m,\ Pr,\ \Theta_w),\qquad b_T = Pe_v/Nu_{0m}(Re_m,\ Pr,\ \Theta_w), \tag{17}$$

where $Pe_v = 2c_{pw}\rho_w V_w r_0/\lambda_m$ is the injection Peclet number; $Nu_{0m}$ ($Re_m$, Pr, $\theta_w$) is the Nusselt number in a pipe with impenetrable walls with variable physical properties. The results of this generalization for $0.3 \leq \theta_w \leq 3$ are shown in Fig. 3; the solid curve corresponds to calculations with constant physical properties. The relation $\Psi_T(b_T)$ for constant properties [15, 16] turns out to be practically universal for all Reynolds numbers and for injection for $Pr \geq 0.7$ is approximated by the formula

$$\Psi_T = \frac{b_{T1}\exp b_{T1}}{\exp b_{T1}-1},\quad b_{T1} = \frac{Pr}{0{,}25+Pr}\,b_T. \tag{18}$$

Thus, if the heat-transfer coefficient is known in a pipe with impenetrable walls for variable properties, the heat-transfer coefficient when there is injection can be calculated with Eq. (18), taking account of relations (17).

## NOTATION

$u_x$, $u_r$, axial and radial velocity components; p, pressure; h, enthalpy; T, temperature; $\rho$, density; $\mu$, viscosity; $\lambda$, thermal conductivity; $c_p$, specific heat; $r_0$, radius of pipe; $V_w$, rate of injection ($V_w < 0$); $\overline{\rho U}$, mass velocity; $M = \rho_w V_w / \overline{\rho U}$, intensity of injection; $q_w$, heat flux density through wall; $\theta_w = T_w/T_m$, temperature factor; $\delta_{ij}$, Kronecker symbol; $\bar{r} = r/r_0$; $\bar{y} = 1 - \bar{r}$; $\bar{\rho} = \rho/\rho_w$; $\bar{\mu} = \mu/\mu_w$; $\bar{\lambda} = \lambda/\lambda_w$; $\bar{c}_p = c_p/c_{pw}$; $R = \rho_w V_w r_0/\mu_w$; $Re_w = 2\overline{\rho U} r_0/\mu_w$; $Re_m = 2\overline{\rho U} r_0/\mu_m$; $Pr = \mu/c_p\lambda$; $K_c = c_{pw}/c_{pm}$; $Q = 2q_w r_0/c_{pw}\mu_w T_w$; $K_p = \rho_w^2 r_0^2/\overline{\rho U}\rho_m \mu_w dp/dx$.
Subscripts: m and w refer to gas parameters determined by the mean mass temperature and the wall temperature.

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